

DEPENDENCE BETWEEN TEMPERATURE VARIABILITY AND AGE-SPECIFIC MORTALITY IN UTTAR PRADESH: A SIMULATION STUDY USING SARIMA AND BIVARIATE PEAKS-OVER-THRESHOLD MODELS

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ABSTRACT

Climate change has increased the frequency and intensity of extreme temperature events, posing substantial risks to public health. This study investigates the association between extreme temperatures and mortality in Uttar Pradesh, India, using simulated monthly temperature and mortality data for the period 2000–2020. The analysis considers four age groups and employs the T90 and T10 temperature indices to represent extreme high- and low-temperature events, respectively. Seasonal Auto-Regressive Integrated Moving Average (SARIMA) models were used to account for temporal and seasonal patterns in the data, while the bivariate Peaks-Over-Threshold (POT) approach was applied to examine extremal dependence through the tail dependence coefficient (χ) and the Pickands dependence function. The results indicate stronger dependence between mortality and extreme low-temperature events than with extreme high-temperature events, although the magnitude of the association varies across age groups. These findings demonstrate the usefulness of combining time-series and extreme value methods for evaluating temperature-related mortality risks under simulated climatic conditions. The proposed framework provides a basis for understanding potential health impacts of future climate variability and can support the development of heat- and cold-related public health adaptation strategies in Uttar Pradesh.

KEYWORDS: Extremal dependence, Bivariate peaks-over-threshold, Extreme temperature and mortality, ARIMA model

INTRODUCTION

In recent decades, climate change has substantially increased the frequency, intensity, and duration of extreme temperature events, leading to growing concerns about their impacts on human health. Both heatwaves and cold spells have been associated with increased mortality, particularly among vulnerable populations such as children, older adults, and individuals with pre-existing health conditions. While numerous studies have examined the effects of extreme high and low temperatures on mortality, relatively few have investigated the impact of the frequency of extreme temperature events and their extremal dependence with mortality. Ignoring the increasing occurrence of repeated heatwaves and cold spells may underestimate the overall health burden associated with climate change.

Uttar Pradesh (UP), India's most populous state, is highly vulnerable to extreme temperatures because of its location in the Indo-Gangetic Plain. During the summer season, particularly under severe heatwave conditions, several districts frequently record some of the highest maximum temperatures in the country, often exceeding 45°C and occasionally reaching 48–49°C. Districts such as Banda and Prayagraj have repeatedly ranked among the hottest locations in India, with the semi-arid Bundelkhand region being especially susceptible due to its rocky terrain, sparse vegetation, low soil moisture, and limited rainfall. Hot, dry winds locally known as the *loo*, together with intense solar radiation and low relative humidity, further intensify thermal stress. Although Uttar Pradesh experiences extremely high summer temperatures, it also undergoes marked seasonal variation, with cold winters and humid monsoon conditions. Historical heatwave events have resulted in substantial heat-related mortality, emphasizing the significant public health burden associated with extreme temperatures. These climatic characteristics make Uttar Pradesh an important region for investigating the health impacts of temperature extremes.

Previous studies have primarily employed time-series regression models, distributed lag models, generalized additive models, and case-crossover designs to quantify the association between temperature and mortality. Although these approaches effectively estimate the average effects of temperature, they may not adequately capture dependence between rare and extreme temperature events and extreme mortality outcomes. Extreme Value Theory (EVT) provides an appropriate statistical framework for modelling such rare events by focusing on the tails of probability distributions. In particular, the Peaks-Over-Threshold (POT) approach models exceedances above selected thresholds using the Generalized Pareto Distribution, enabling more reliable estimation of extreme behavior. However, relatively few studies have applied EVT to examine the extremal dependence between temperature extremes and mortality in Uttar Pradesh, highlighting an important research gap.

In this study, Seasonal Auto-Regressive Integrated Moving Average (SARIMA) models are employed to remove temporal trends and seasonal variation from the monthly temperature and mortality series. The resulting residuals are analyzed using a bivariate Peaks-Over-Threshold (POT) framework to investigate the extremal dependence between temperature extremes and mortality through the tail dependence coefficient (χ) and the Pickands dependence function. Unlike conventional correlation analyses, this approach specifically examines whether unusually high or low temperatures are associated with extreme mortality events.

Using simulated monthly temperature and mortality data for Uttar Pradesh covering the period 2000–2020, this study evaluates the relationship between the extreme temperature indices T90 and T10 and mortality across different age groups. The findings are expected to improve understanding of temperature-related health risks under extreme climatic conditions and provide methodological guidance for climate-health research, public health preparedness, and climate adaptation strategies in Uttar Pradesh.

The remainder of this paper is organized as follows. Section 2 describes the study data and presents the exploratory data analysis. Section 3 outlines the methodology, including the Seasonal Auto-Regressive Integrated Moving Average (SARIMA) model and the bivariate Peaks-Over-Threshold (POT) approach within the framework of Extreme Value Theory (EVT). Section 4 presents the empirical results, followed by a discussion of the findings in Section 5. Finally, Section 6 summarizes the main conclusions, highlights the study's contributions, and discusses directions for future research.

DATA DESCRIPTION

This research was conducted based on a simulation study inspired by the climatic characteristics of Uttar Pradesh, India. It aims to investigate the association between monthly age-specific mortality and temperature variability during the period 2000–2020

using simulated temperature and mortality data. Extreme temperature conditions are represented by the T90 and T10 indices, while Seasonal Auto-Regressive Integrated Moving Average (SARIMA) models and the bivariate Peaks-Over-Threshold (POT) approach are employed to examine the extremal dependence between temperature variability and mortality across different age groups. The findings are expected to provide methodological insights into temperature-related health risks and contribute to climate-health research and public health adaptation strategies.

Two temperature indices, T90 and T10, were selected to represent extreme temperature conditions. The T90 index measures the frequency of temperatures exceeding the 90th percentile, whereas the T10 index represents the frequency of temperatures falling below the 10th percentile, based on the reference period 2000–2020. The study area comprises the state of Uttar Pradesh, India, extending approximately from 23.87°N to 30.40°N latitude and 77.05°E to 84.65°E longitude. Temperature values were represented on gridded data with a spatial resolution of $2.5^\circ \times 2.5^\circ$, covering the entire study region. In this study, the monthly T90 and T10 indices were analysed using their unsmoothed and unstandardized values.

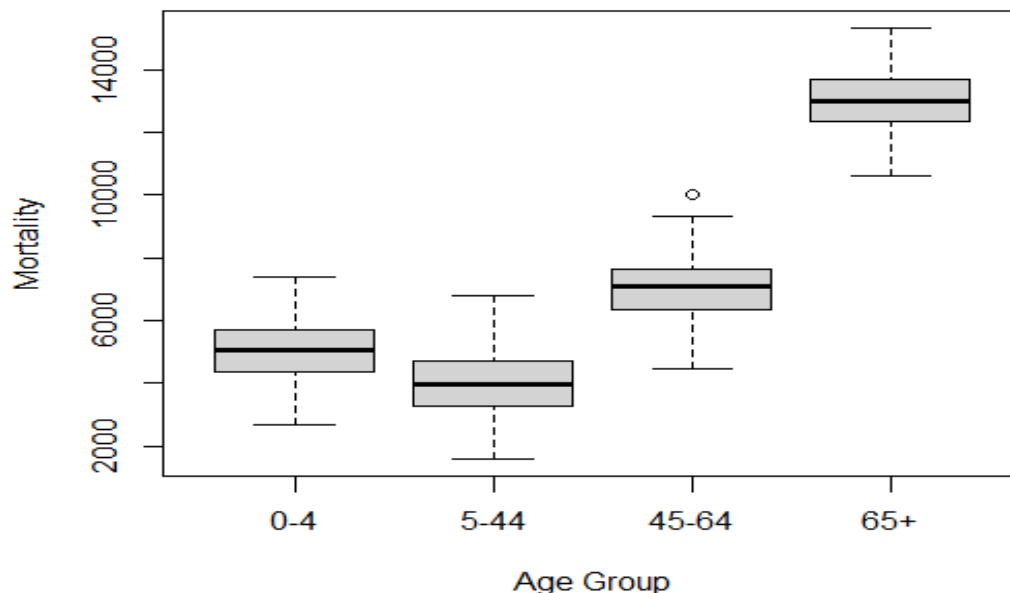


Figure 1. Boxplot of monthly mortality by age group in Uttar Pradesh (2000–2020)

The boxplot of monthly mortality by age group indicates substantial differences in mortality levels across age categories. The Age 65 plus group exhibits the highest mortality distribution, suggesting that older adults experience the greatest mortality burden. The Age 45 to 54 group shows the second highest mortality, while the Age 0 to 4 and Age 5 to 44 groups have comparatively lower mortality levels. The spread of the boxplots suggests variation in monthly mortality over time, with older age groups showing greater fluctuations. These findings indicate that mortality risk is age-dependent and that elderly populations may be more vulnerable to adverse environmental conditions, including extreme temperatures.

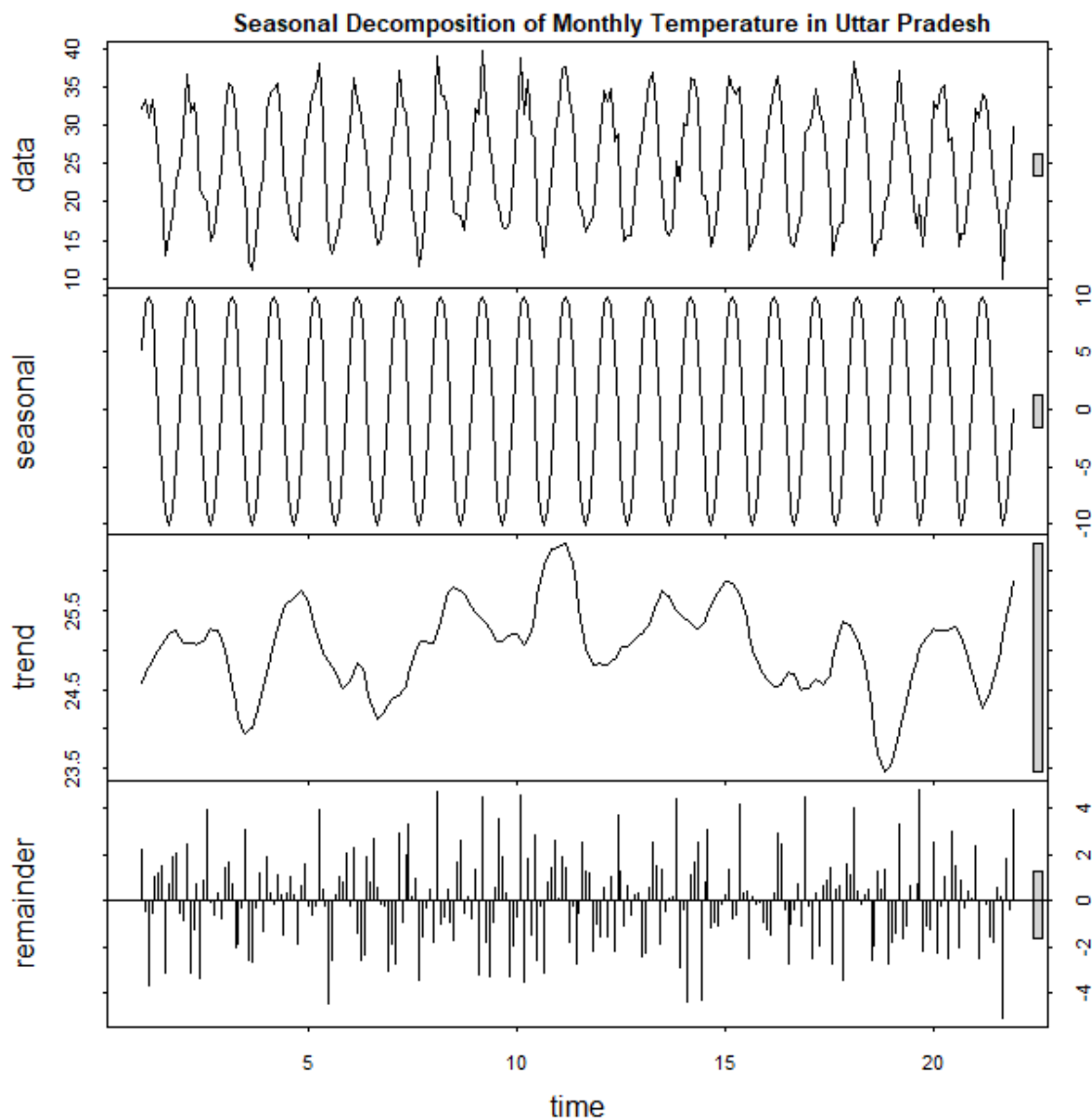


Figure 2. Seasonal decomposition of monthly mortality in Uttar Pradesh (2000–2020).

Figure 2 presents the seasonal decomposition of the monthly mortality series in Uttar Pradesh from 2000 to 2020 into four components: the observed data, seasonal, trend, and remainder. The observed series exhibits a clear upward trend together with pronounced seasonal fluctuations throughout the study period. The seasonal component demonstrates a regular annual cycle, indicating that mortality follows a consistent seasonal pattern each year. The trend component reveals a gradual increase in mortality over time, suggesting a long-term upward movement in the series. The remainder component fluctuates randomly around zero without any obvious systematic pattern, indicating that the major trend and seasonal variations have been effectively captured by the decomposition.

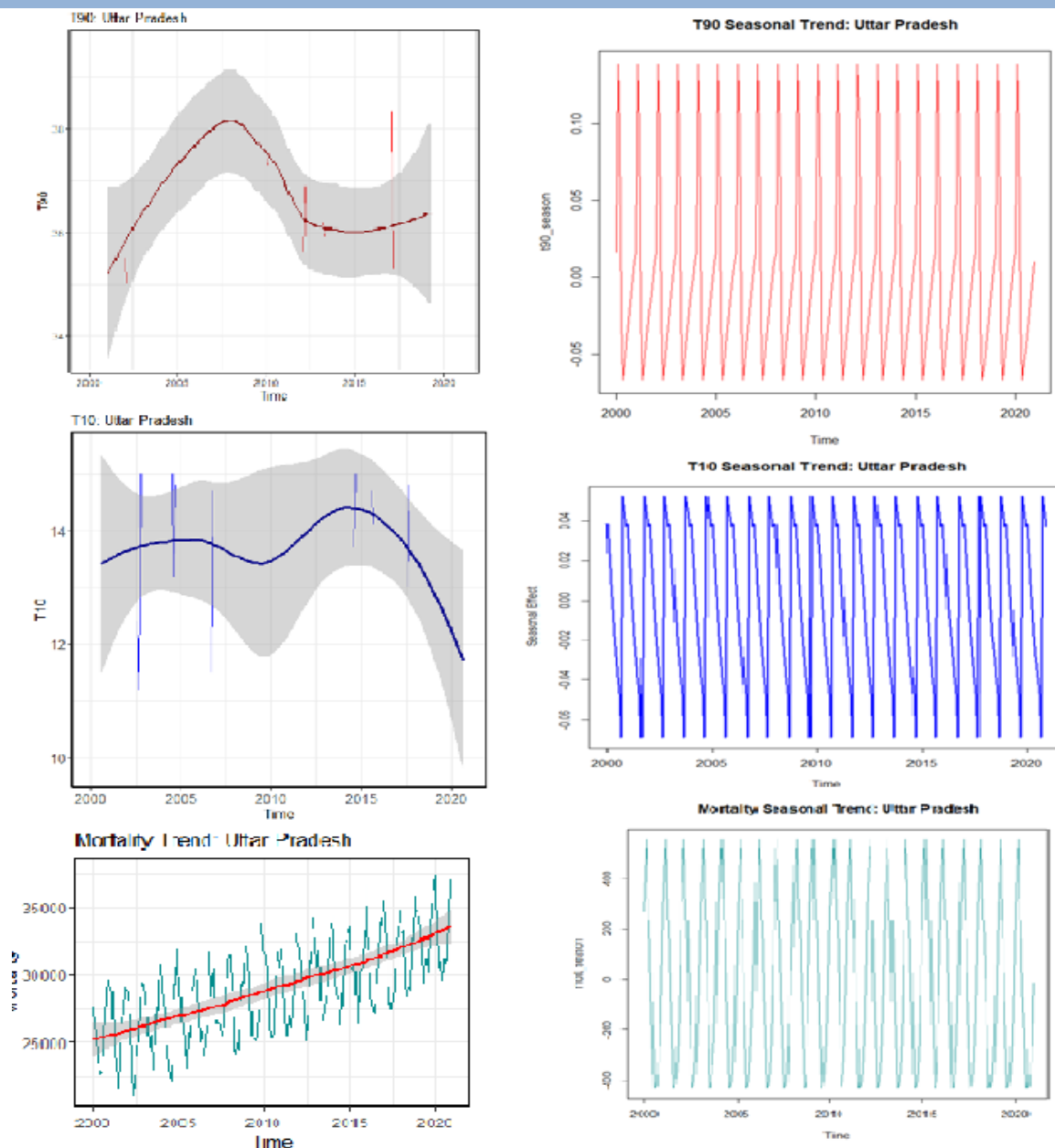


Figure 3. Temporal trends and seasonal patterns of T90, T10, and monthly mortality in Uttar Pradesh (2000–2020).

Figure 3 illustrates the temporal trends and seasonal patterns of the T90 index (extreme high temperature), the T10 index (extreme low temperature), and monthly mortality in Uttar Pradesh during the period 2000–2020. The left panels display the smoothed long-term trends together with their corresponding confidence intervals, while the right panels present the seasonal components extracted from the time series. The T90 index exhibits noticeable interannual variability, with an increasing trend during the early years followed by a decline and slight recovery toward the end of the study period. In contrast, the T10 index remains relatively stable during the first half of the study period, increases moderately after 2010, and declines slightly in the later years. Monthly mortality shows a clear upward long-term trend throughout the study period, indicating a gradual increase in mortality over time. The seasonal components of T90, T10, and mortality exhibit regular annual cycles, demonstrating strong and persistent seasonality. These findings indicate that both temperature indices and mortality possess pronounced seasonal characteristics, supporting the application of Seasonal Auto-Regressive Integrated Moving Average (SARIMA) models for subsequent time-series analysis.

METHODOLOGY

In this section, we introduce seasonal ARIMA models in “ARIMA model” section to adjust the seasonality and tendency of the data involved. Then, we introduce the extreme value theory, including the details of the bivariate POT method in “Extreme value theory” section.

ARIMA model

Seasonal variation of mortality was discovered with association with socioeconomic status and outdoor temperature due to the nature of the climate change and health vulnerability. The seasonality usually causes the series to be non-stationary. To remove the possible seasonality and tendency of the data, the seasonal ARIMA model is applied. It is one of the most commonly used statistical models to deal with non-stationary time series, with three key components: auto-regression (AR), integration (I) for non-stationary data, and current and previous residual series moving average (MA). ARIMA (p, d, q) with orders p, d, q , is defined by

$$y_t = c + \phi_1 y_{t-1} + \dots + \phi_p y_{t-p} + \theta_1 \varepsilon_{t-1} + \dots + \theta_q \varepsilon_{t-q} + \varepsilon_t,$$

where y_t is the differenced data, c is the constant term, $\phi_1, \dots, \phi_p$ and $\theta_1, \dots, \theta_q$ are parameters, ε_t is the white noise with zero mean and constant variance, and p, d and q denote the order of the AR model, the order of differencing, and the order of the MA model, respectively. AR and MA are two widely used linear models that work on stationary time series. A seasonal ARIMA model incorporates both non-seasonal and additional seasonal factors in the ARIMA model, denoted by

$$\text{ARIMA}(p, d, q) \times (P, D, Q)S,$$

where P, D and Q have identical meanings that are in the non-seasonal part of the model, but involve the time span of the seasonality S .

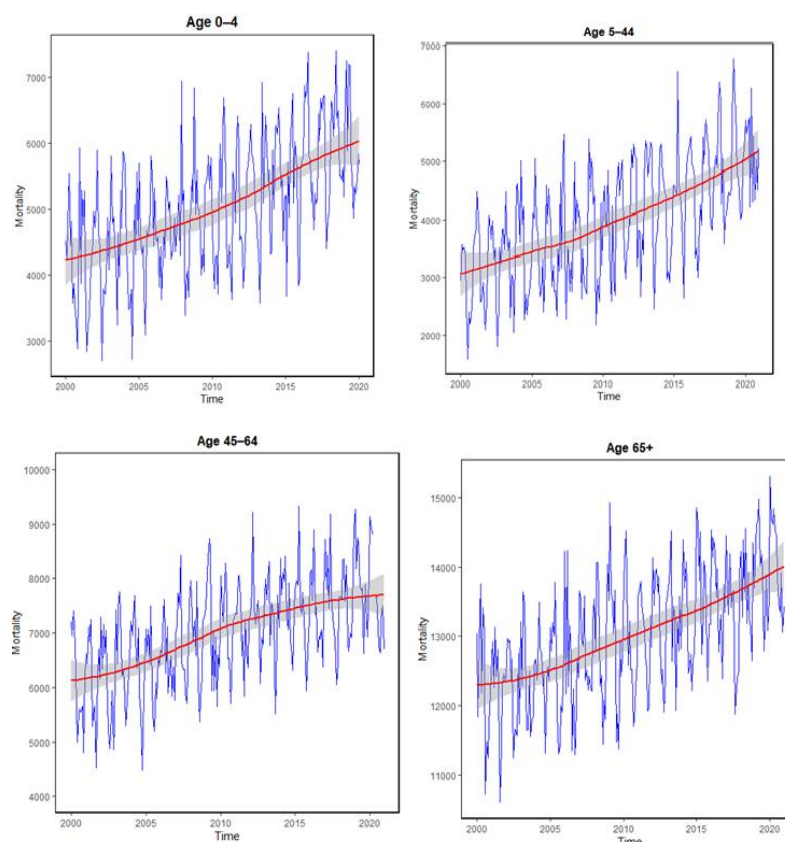


Figure 4. Age wise mortality in Uttar Pradesh (2000 -2020)

In our case, we set S to be 12 for monthly data. The residuals of seasonal ARIMA models are expected to be “white noise” (i.e., independent and identically distributed),

which will be confirmed by the Box-Pierce and Ljung-Box tests at 5% significance for the optimal ARIMA model for T90, T10 and mortality in terms of the Akaike information criterion (AIC). The general process of fitting seasonal ARIMA models is based on the Box-Jenkins method. When dealing with time series data that exhibit strong seasonality and are non-stationary, we first perform differencing to make the data stationary, and then apply both Box-Pierce and Ljung-Box tests at 5% significance, to examine whether data behaves like white noise. In particular, we utilize the `auto.arima()` function from the `forecast` R-package to find the optimal model with the lowest AIC. Then, we obtain the residuals of the optimal model by `arma()` function in `stats` package, and apply both Box-Pierce and Ljung-Box tests with `Box.test()` function. If the residuals fail to meet the criterion for white noise (i.e., p-value is less than 0.05), we proceed to fit a generalized auto-regressive conditional heteroscedasticity (GARCH) model with `garch()` function in `tseries` package. Again, we perform the aforementioned two tests on the residuals of the GARCH model to ensure that they exhibit characteristics of white noise.

Extreme value theory

Extreme value theory (EVT) is widely applied in the study of rare events with extreme influence on economics, finance, environment and public health [15]. Suppose that $X_1, X_2, \dots, X_n$ is a random sample from parent $X \sim F(x)$, i.e., X_i 's are independent and identically distributed with common distribution function (df) $F(x)$. Given a large threshold u , the distribution $F_u(y)$ of the excess $Y_{[u]} = X - u \mid X > u$, is thus given by

$$F_u(y) = P\{X - u \leq y \mid X > u\} = \frac{P\{u < X \leq y + u\}}{P\{X > u\}} = \frac{F(y + u) - F(u)}{1 - F(u)}$$

The $F_u(y)$ can be approximated by generalized Pareto (GP) distribution $G_{\xi, \sigma}(y) = 1 - (1 + \xi y / \sigma)^{-1/\xi}$, $y > 0$ for sufficiently high threshold. Therefore, the tail distribution function $\bar{F}(x) = 1 - F(x)$ of X can be approximated by

$$\bar{F}(x) = \zeta_u \bar{F}_u(x - u) \approx \zeta_u \bar{G}_{\xi, \sigma}(x - u), \quad x > u \quad (1)$$

Here $\zeta_u = \bar{F}_u$, and $\xi \in \mathbb{R}$ and $\sigma > 0$ are the shape/tail and scale parameters of GP distribution, respectively. In practice, the exceedance probability $F(x)$ gives insights into the potential risk, and a larger tail risk is indicated by a larger tail index ξ . Its estimate can be obtained through the extrapolation approach via Equation (1) to get the approximated tail probability of the GP model using the maximum likelihood estimation of ξ , σ based on excesses $(x_{(i)} - u)$'s with $x_{(1)} \geq \dots \geq x_{(n_u)}$ exceeding the threshold u and the estimate of ζ_u as n_u/n . Theoretically, the threshold u can be determined by minimizing the mean square error of the Hill estimator of ξ , balancing the model bias and variance. A common graphical approach in the determination of the threshold is to check both the linearity of the empirical mean excess function and the stability estimation plots of both scale and shape parameters.

Bivariate POT method

Many problems involving extreme events are inherently multivariate. A fundamental issue thus arising is how extremes in one variable relate to those in another. Dependence occurs if the process is studied at neighboring spatial locations during its temporal evolution or shares common meteorological conditions. The study of multivariate extremes splits apart into two steps: the marginal analysis first and then the dependence measures, which are commonly supposed to be marginally invariant. Commonly, one may employ marginal tail information to transform the marginal distribution to a common scale, e.g., unit Fréchet distribution. Let (X_i, Y_i) be a random sample of size n with common distribution function (d.f.) $F(x, y)$. Assume its marginal F_X and F_Y satisfy Equation (1) on regions of the form $x > u_x$ and $y > u_y$ for large enough thresholds u_x and u_y , with respective parameter sets $(\xi_x, \sigma_x, \zeta_x)$ and $(\xi_y, \sigma_y, \zeta_y)$. Denote by

$$\tilde{F}(\tilde{x}, \tilde{y}) = F(x, y), \quad x > u_x, \quad y > u_y$$

Suppose that F is in the max-domain of attraction of a bivariate extreme value

distribution, i.e., the normalized component-wise maxima of observation from F follow asymptotically a bivariate extreme value distribution where $\tilde{x}$ and $\tilde{y}$ are defined in terms of x and y and G is a bivariate with unit Fréchet margins. Note that G has no parametric form. In our context, we take a flexible logistic model for our bivariate statistical analysis is given by

$$G(\tilde{x}, \tilde{y}) = \exp -(\tilde{x}^{-1/\alpha} + \tilde{y}^{-1/\alpha})^\alpha$$

with dependence parameter $\alpha \in (0, 1]$. The limiting case of $\alpha \rightarrow 0$ corresponds to the variables being total dependent. That is

$$G(\tilde{x}, \tilde{y}) \rightarrow \exp -\max(\tilde{x}^{-1}, \tilde{y}^{-1}) , \text{ as } \alpha \rightarrow 0.$$

As increases the dependence becomes weak, and when $\alpha = 1$, the variables are independent.

Hence, the bivariate logistic model covers all levels of dependence, from independence to perfect dependence. One of the most useful alternative approaches to describe the dependence structure of G is extreme tail index $\chi \in [0, 1]$, giving a rough but representative picture of the full dependence structure. The extreme tail index is a limiting measure of the tendency for one variable to be large conditional on the other variable being large i.e.,

$$\chi = \lim_{u \rightarrow 1} P\{F_Y(Y) > u \mid F_X(X) > u\},$$

where F_X and F_Y are the marginal d.f.s of X and Y . We see that, the index χ describes the likelihood that the quantity X (here the excess high or low temperature frequency) becomes so large as to cause Y (here the excess mortality) to experience such a tail event at least as severe (in quantile terms) as X , ranging in $[0, 1]$. In particular, the variables are said to be asymptotically independent when $\chi = 0$, while $\chi = 1$ corresponds to the total dependence. Another alternative approach for extremal dependence is to develop functions that give a complete characterization of the extremal dependence, like the spectral distribution function, the Pickands dependence function [36] or the stable tail dependence function [37]. These functions can be seen as the analogues of copulas in classical statistics. In this paper, we will utilize the Pickands dependence function

$$A(\omega) = -\log G(1/(1 - \omega), 1/\omega) : [0, 1] \rightarrow [0, 1],$$

Note that the lower and upper bounds of the Pickands dependence function above correspond to the total dependence and independence, respectively. Therefore, we can show graphically the dependence by examining the closeness of the lower bound of the Pickands dependence function. With white-noise residuals obtained from the ARIMA/ ARIMA-GARCH models, we employ the bivariate POT method with implementation in the POT R-package [38]. The marginal thresholds (u_x, u_y) can be selected by empirical mean residual life plots and scale/shape parameters' stability plots. These plots are generated using the `mrlplot()` and `tcplot()` functions, respectively. Then we proceed to fit the joint residuals by bivariate GP distribution with the logistic dependence structure, implemented by `fitbvdpd()` function. The extreme tail index χ is subsequently calculated based on the reported α in the bivariate logistic model, and the Pickands dependence function is visualized by the `pickdep()` function.

RESULTS

The bivariate POT method requires data to be independent and identically distributed, thus the ARIMA model is applied to adjust the seasonality, trends and non-stationarity from temperature frequency and mortality time series. We first fit monthly T90, T10 and mortality data by seasonal ARIMA models and perform the BoxPierce and Ljung-Box tests on the residuals of the optimal ARIMA model in terms of AIC values. We then present the main results of bivariate POT analysis for residuals of T90, T10 and mortality.

Variable	Non-seasonal ARIMA (p, d, q)	Seasonal ARIMA (P, D, Q)
T90	(4, 0, 0)	(0, 0, 0)
T10	(2, 0, 0)	(0, 0, 0)
Mortality (Overall)	(0, 0, 0)	(2, 0, 0)
Age 0–4 years	(0, 0, 0)	(2, 1, 0)
Age 5–44 years	(0, 0, 0)	(2, 1, 0)
Age 45–64 years	(0, 0, 0)	(2, 1, 0)
Age 65 years and above	(1, 0, 0)	(2, 1, 0)

Table 1. Selected SARIMA models for temperature indices and age-specific mortality in Uttar Pradesh (2000–2020)

Result of ARIMA model

Table 1 presents the optimal SARIMA models selected using the `auto.arima()` procedure based on the Akaike Information Criterion (AIC). The T90 series was best fitted by an ARIMA (4,0,0) model without seasonal components, while the T10 series followed an ARIMA (2,0,0) model. The overall mortality series was adequately described by a seasonal ARIMA (0,0,0)(2,0,0)₁₂ model. For the age-specific mortality series (0–4, 5–44, and 45–64 years), the selected model was ARIMA (0,0,0) (2,1,0)₁₂ with drift, whereas the mortality series for individuals aged 65 years and above was best represented by ARIMA(1,0,0)(2,1,0)₁₂ with drift. These results indicate that seasonal autoregressive effects play a dominant role in explaining the temporal dynamics of mortality across different age groups.

Results of bivariate POT models

To investigate the extremal dependence between temperature variability and mortality, the bivariate Peaks-Over-Threshold (POT) model was applied separately to the T90 (extreme high temperature) and T10 (extreme low temperature) indices for four age groups (0–4, 5–44, 45–64, and 65 years and above). The extremal dependence structure was evaluated using the estimated Pickands dependence function, $A(\omega)$. Under complete independence, the Pickands dependence function follows the upper boundary $A(\omega)=1$, whereas stronger extremal dependence is indicated by greater deviation from this boundary toward the lower theoretical limit.

T10 and Mortality

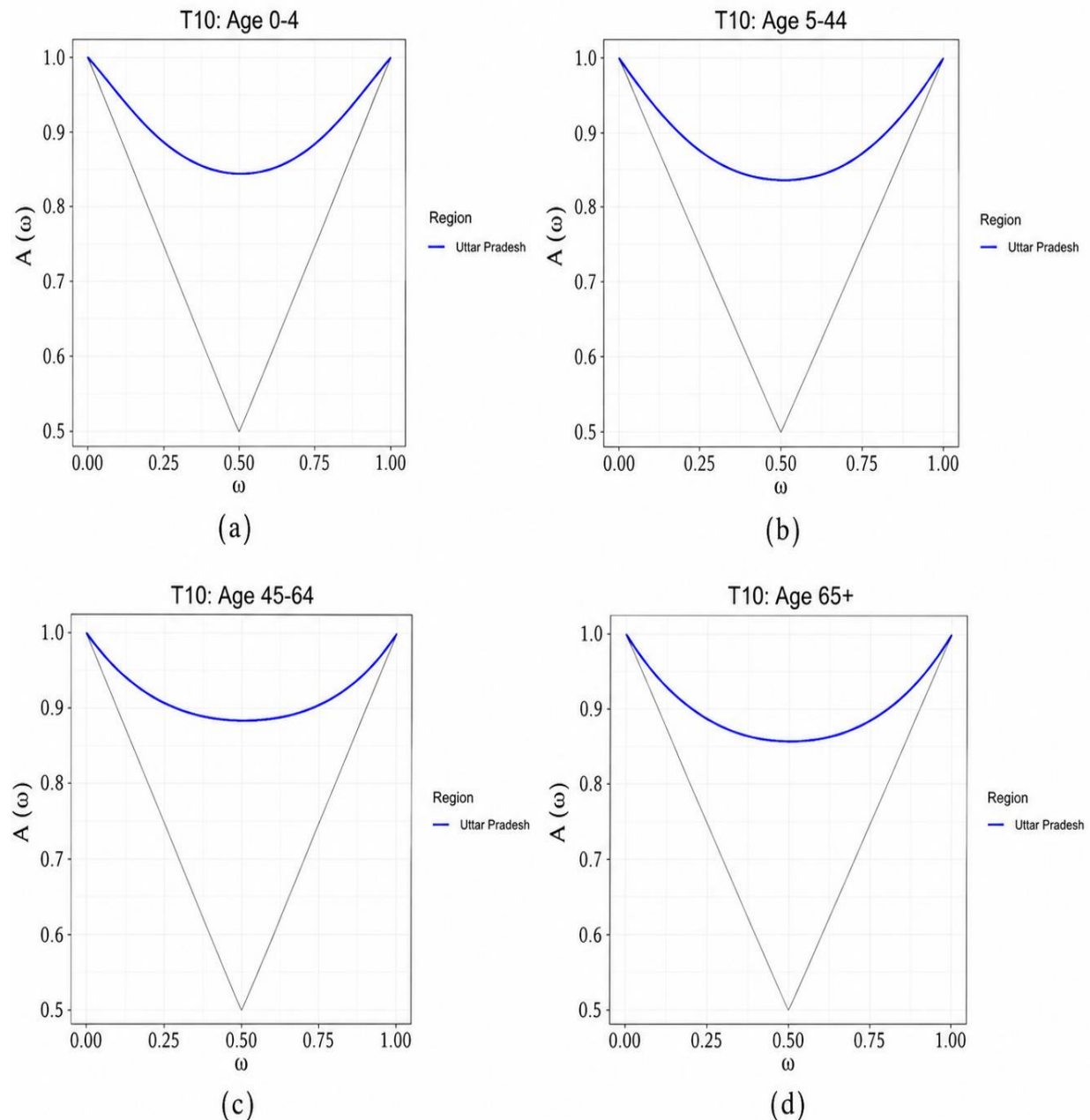


Figure 5. T10 and Mortality of Uttar Pradesh

Figure 5 presents the estimated Pickands dependence functions for the association between the T10 index and age-specific mortality in Uttar Pradesh. The estimated curves exhibit a smooth convex shape for all four age groups, lying below the independence boundary but above the theoretical lower bound. This pattern suggests the presence of moderate positive extremal dependence between extreme low-temperature events and mortality. The curves for the 5–44 and 45–64 age groups show slightly greater deviation from the independence boundary than those of the 0–4 and 65+ groups, indicating relatively stronger dependence between cold-temperature extremes and mortality among the middle-aged population. Overall, the results suggest that unusually frequent cold-temperature events are associated with increased mortality across all age groups.

T90 and Mortality

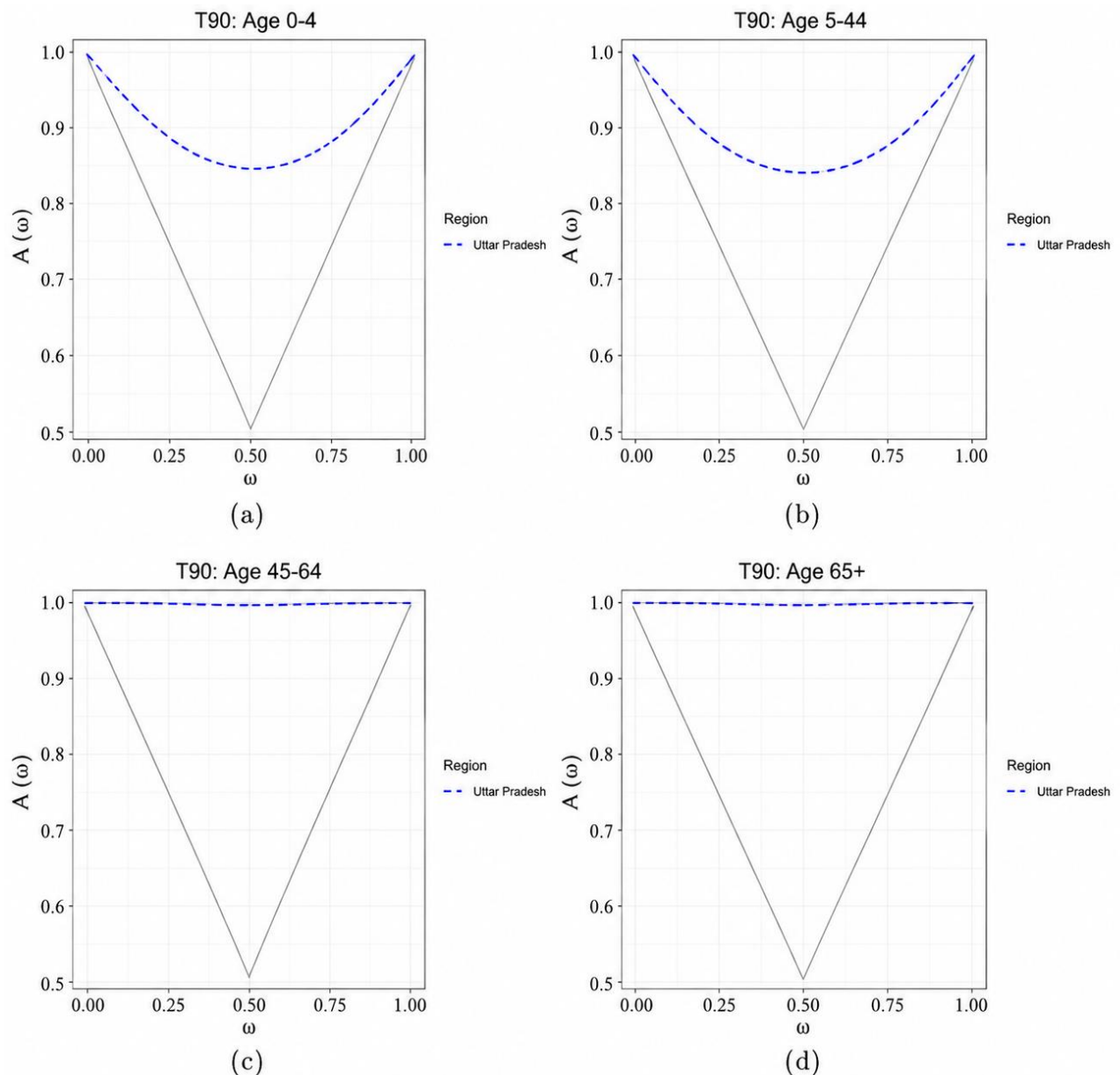


Figure 6. T90 and Mortality of Uttar Pradesh

Figure 6 illustrates the estimated Pickands dependence functions for the relationship between the T90 index and age-specific mortality. For the 0–4 and 5–44 age groups, the estimated curves lie slightly below the independence boundary, indicating weak to moderate extremal dependence between extreme high temperatures and mortality. In contrast, the curves for the 45–64 and 65+ age groups remain very close to the independence boundary, suggesting little or no extremal dependence between frequent extreme heat events and mortality. These findings indicate that the association between high-temperature extremes and mortality is generally weaker than that observed for extreme low temperatures.

Comparing Figures 5 and 6 reveals that the extremal dependence between mortality and T10 is generally stronger than that between mortality and T90 across all age groups. This suggests that, under the simulated climatic conditions of Uttar Pradesh, unusually frequent

cold-temperature events may pose a greater mortality risk than extreme heat events. Although the magnitude of dependence varies among age groups, the bivariate POT analysis demonstrates that the proposed methodology effectively characterizes the joint behaviour of extreme temperature events and mortality.

DISCUSSION

Uttar Pradesh has a subtropical climate with very hot summers and cool winters. Extreme heat events are more common than extreme cold, particularly during the pre-monsoon season. Several districts frequently experience temperatures above 45°C during severe heatwaves. Cold spells in December and January may also increase mortality among vulnerable populations. The bivariate Peaks-Over-Threshold (POT) analysis indicates that the dependence between temperature extremes and mortality varies across age groups. The estimated Pickands dependence functions show that **T10** (extreme low temperature) has a stronger association with mortality than **T90** (extreme high temperature). The T10 curves exhibit moderate extremal dependence, whereas the T90 curves remain close to the independence boundary, indicating weaker dependence.

Children (0–4 years) and adults aged 5–44 years show relatively stronger dependence between temperature extremes and mortality than older age groups. These findings suggest that the health effects of extreme temperatures differ by age. Overall, the study demonstrates that the bivariate POT framework is effective for assessing the relationship between rare temperature extremes and mortality. Although based on simulated data, the methodology provides a useful foundation for future studies using observed climate and health records in Uttar Pradesh. The results can support the development of heat–health warning systems, climate adaptation strategies, and public health policies to reduce climate-related mortality.

CONCLUSION

This study developed a simulation-based framework to examine the relationship between extreme temperature variability and age-specific mortality in Uttar Pradesh during 2000–2020. Simulated monthly temperature and mortality data were analysed using SARIMA models and the bivariate Peaks-Over-Threshold (POT) approach within the Extreme Value Theory (EVT) framework. The methodology successfully captured temporal patterns and estimated the dependence between temperature extremes and mortality. The results indicated that the T10 index (extreme low temperature) showed stronger extremal dependence with mortality than the T90 index (extreme high temperature).

The strength of this relationship varied across different age groups, highlighting age-specific vulnerability to temperature extremes. The study demonstrates that the bivariate POT approach is an effective tool for analysing joint extreme climate and health events. Although the analysis was based on simulated data, the proposed framework provides a strong methodological foundation for future research using observed temperature and mortality data. The findings can support the development of heat–health warning systems, climate adaptation measures, and evidence-based public health planning to reduce climate-related mortality in Uttar Pradesh.

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