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AN INVESTIGATION ON THE INTER RELATIONS BETWEEN FUZZY ROUGH UTRA TM CONNECTED SYSTEM AND OTHER FUZZY ROUGH ALGEBRAIC TM SYSTEM

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Abstract

This study presents and investigates fuzzy rough utra TM connected spaces, a novel type of fuzzy rough algebraic TM system. It is established that fuzzy rough algebraic TM systems' hypo connectedness and fuzzy rough utra TM connected spaces are distinct concepts, and that fuzzy rough algebraic open hereditarily irresolvable system are fuzzy rough utra TM connected but not hypo connected. Examples are provided to prove the concept of inter relations.

Keywords: Fuzzy rough Utra TM connected system, Fuzzy rough hypo TM connected system, Fuzzy rough algebraic TM Brown system, Fuzzy rough TM dense, Fuzzy rough nowhere TM dense, Fuzzy rough somewhere TM dense.

Introduction

A valuable notion in Mathematics and Computer Science, Fuzzy Connectedness offers a number of intriguing characteristics. It can be used, for instance, to define fuzzy connected components, the maximal fuzzy connected subsets of a given fuzzy space. It can also be used to segment images based on fuzzy connectivity in image processing and computer vision. The concept of fuzzy ultra-connected spaces and the relation between topological spaces and other connected spaces were discussed by Thangaraj G and Ponnusamy M [5]. This paper deals with the relation between different connected spaces in fuzzy rough algebraic TM system.

1.2 PRELIMINARIES

Definition 1.2.1 [2]

A fuzzy set λ in a fuzzy topological space (X, T) is called fuzzy dense if there exists no fuzzy closed set μ in (X, T) such that $\lambda < \mu < 1$. That is, $cl\,\lambda = 1$, in (X, T).

Definition 1.2.2 [2]

A fuzzy set λ in a fuzzy topological space (X, T) is called fuzzy nowhere dense if there exists no non-zero fuzzy open set μ in (X, T) such that $\mu \subset cl\,(\lambda)$. That is $int(cl(\lambda))$ = 0, in (X, T).

Definition 1.2.3 [4]

A fuzzy set α in a fuzzy topological space (X, τ) is called a fuzzy somewhere dense set if there exists a non-zero fuzzy open set β in (X, τ) such that $\beta \leq cl(\alpha)$. That is, $intcl(\alpha) \neq 0$ in (X, τ).

Definition 1.2.4 [5]

A fuzzy topological space is said to be a fuzzy hyper connected space if every non-zero fuzzy open subset of (X, τ) is fuzzy dense in (X, τ).

Definition 1.2.5 [6]

A fuzzy fuzzy topological space is said to be a fuzzy Brown space if for any two non-zero fuzzy open sets α and β in (X, τ), $cl(\alpha) \nleq 1 - cl(\beta)$ in (X, τ).

1.3 Fuzzy Rough Utra TM Connected System

Definition 1.3.1

Let X be a rough set. Then X is said to be a rough algebraic system, if X_L and X_U are TM algebras with $X_L \subset X_U$.

Definition 1.3.2

Let L be a lattice. For any rough algebraic TM system X and any fuzzy rough set, A is said to be fuzzy rough algebraic if A_L is a mapping from $X_L \to I$ and A_U is a mapping from $X_U \to I$ with $A_L(x) \leq A_U(x)$ for every $x \in X_U$. The collection of all such sets in X is denoted by $\mathcal{FR}(X_{\mathcal{TM}})$.

Definition 1.3.3

A fuzzy rough TM algebra A in a fuzzy rough algebraic TM system (X, TM) is said to be a fuzzy rough TM dense if there exists no fuzzy rough closed algebraic B in (X, TM) such that $A \subseteq B \subseteq \tilde{1}$. That is $\mathcal{F}_{\mathcal{RTM}}cl(A) = \tilde{1}$ in (X, TM).

Definition 1.3.4

A fuzzy rough algebraic A in a fuzzy rough algebraic TM system (X, TM) is called a fuzzy rough nowhere TM dense algebra if there exists a non-zero fuzzy rough open algebraic D in (X, TM) such that $D \subseteq \mathcal{F}_{\mathcal{RTM}}cl(A)$. That is, $\mathcal{F}_{\mathcal{RTM}}intcl(A) = \tilde{0}$ in (X, TM).

Definition 1.3.5

A fuzzy rough algebraic A in a fuzzy rough algebraic TM system (X, TM) is called a fuzzy rough somewhere TM dense algebra if there exists a non-zero fuzzy rough open algebraic B in (X, TM) such that $B \subseteq \mathcal{F}_{\mathcal{RTM}}cl(A)$. That is, $\mathcal{F}_{\mathcal{RTM}}intcl(A) \neq \tilde{0}$ in (X, TM).

Definition 1.3.6

Let (X, TM) be a fuzzy rough algebraic TM system. Then (X, TM) is said to be the fuzzy rough utra TM connected system if whenever A', B' are two non-zero fuzzy rough closed algebraic in (X, TM), $A' \nsubseteq B$, in (X, TM).

Example 1.3.1

Let $U = \{a, b, c\}$ and let $X_L = \{a\}$ and $X_U = \{a, c\}$ with $X_L \subset X_U$.

Then the Boolean algebra is $\mathcal{B} = \{\tilde{0}, \{a\}, \{b\}, \{b, c\}, \{a, c\}, \{a, b\}, \{a, b, c\}\}$.

Define $A_L(x): X_L \to I \Rightarrow A_L(a) = 0.3$

$A_U(x): X_U \to I \Rightarrow A_U(a) = 0.5$ and $A_U(c) = 0.3$

and define $B_L(x): X_L \to I \Rightarrow B_L(a) = 0.2$

$B_U(x): X_U \to I \Rightarrow B_U(a) = 0.2$ and $B_U(c) = 0.3$

Then $A = \left\{\left(\frac{a}{0.3}\right), \left(\frac{a}{0.5}, \frac{c}{0.3}\right)\right\}$ and $B = \left\{\left(\frac{a}{0.2}\right), \left(\frac{a}{0.2}, \frac{c}{0.3}\right)\right\}$ the fuzzy rough algebraic of (X, TM). Therefore, the fuzzy rough algebraic TM system is $\{\tilde{0}, \tilde{1}, A, B\}$. The fuzzy rough algebraic closed A' and B' is given by $A' = \left\{\left(\frac{a}{0.5}\right), \left(\frac{a}{0.7}, \frac{c}{1}\right)\right\}$ and $B' = \left\{\left(\frac{a}{0.8}\right), \left(\frac{a}{0.8}, \frac{c}{1}\right)\right\}$. Then by computation it is found that $A' \nsubseteq B$. Hence (X, TM) is a fuzzy utra connected space.

Example 1.3.2

Let $U = \{a, b, c\}$ and let $X_L = \{a\}$ and $X_U = \{a, c\}$ with $X_L \subset X_U$.

Then the Boolean algebra is $\mathcal{B} = \{\tilde{0}, \{a\}, \{b\}, \{b, c\}, \{a, c\}, \{a, b\}, \{a, b, c\}\}$.

Let $A = \left\{\left(\frac{a}{0.9}\right), \left(\frac{a}{0.9}, \frac{c}{0.1}\right)\right\}$ and $B = \left\{\left(\frac{a}{0.3}\right), \left(\frac{a}{0.4}, \frac{c}{0.3}\right)\right\}$ be the fuzzy rough algebraic of (X, TM). Therefore, the fuzzy rough algebraic TM system is $\{\tilde{0}, \tilde{1}, A, B\}$. The fuzzy rough algebraic closed A' and B' is given by $A' = \left\{\left(\frac{a}{0.1}\right), \left(\frac{a}{0.1}, \frac{c}{1}\right)\right\}$ and $B' = \left\{\left(\frac{a}{0.6}\right), \left(\frac{a}{0.7}, \frac{c}{1}\right)\right\}$. By computation it is found that $A' \subseteq B$. Hence (X, TM) is not a fuzzy utra connected space.

Proposition 1.3.1

If D and E are non – zero fuzzy rough algebraic open in a fuzzy rough utra TM connected system (X, TM), then $\mathcal{F}_{\mathcal{RTM}}cl(D) \nsubseteq \tilde{1} - \mathcal{F}_{\mathcal{RTM}}cl(E)$ in (X, TM).

Proof

Let D and E be non – zero fuzzy rough algebraic open in (X, TM). Then, $\mathcal{F}_{\mathcal{RTM}}cl(D)$ and $\mathcal{F}_{\mathcal{RTM}}cl(E)$ are fuzzy rough regular A fuzzy rough algebraic closed in (X, TM) and thus fuzzy rough algebraic closed in (X, TM). Since (X, TM) is a fuzzy rough utra TM connected system,

$$\mathcal{F}_{\mathcal{RTM}}cl(D) \nsubseteq \tilde{1} - \mathcal{F}_{\mathcal{RTM}}cl(E) \text{ in } (X, TM).$$

Proposition 1.3.2

If B and G are non – zero fuzzy rough TM regular closed in a fuzzy rough utra TM connected system (X, TM), then $B \nsubseteq \tilde{1} - G$ in (X, TM).

Proof

Let B and G are non – zero fuzzy rough TM regular closed. Since fuzzy rough regular A fuzzy rough algebraic closed are fuzzy rough closed algebraic in fuzzy rough algebraic TM system, B and G are fuzzy rough regular A fuzzy rough algebraic closed in the fuzzy rough utra TM connected system (X, TM), $B \nsubseteq \tilde{1} - G$ in (X, TM).

Remark 1.3.1

The following result can be obtained from the *Proposition 1.3.1***.** In a fuzzy rough utra TM connected system, there are no disjoint fuzzy rough regular A fuzzy rough algebraic closed.

For if B and G are any two disjoint fuzzy rough regular A fuzzy rough algebraic closed in a fuzzy rough utra TM connected system (X, TM), then $B \cap G = \tilde{0}$ and this implies that $B \nsubseteq \tilde{1} - G$ in (X, TM), which is contradiction by *Proposition 1.3.1***.** Thus, there are no disjoint fuzzy rough regular TM closed in fuzzy rough utra TM connected system.

Proposition 1.3.3

If a fuzzy rough algebraic TM system is a fuzzy rough utra TM connected system, then there are no disjoint fuzzy rough algebraic TM closed in (X, TM).

Proof

Suppose that D and E are fuzzy rough algebraic TM closed in (X, TM) such that $D \cap E = 0$. This implies that $D \subseteq \tilde{1} - E$ in (X, TM). But this is a contradiction to being a fuzzy rough utra TM connected system in which $D \nsubseteq \tilde{1} - E$, for any two fuzzy rough algebraic closed D and E in (X, TM). Hence there are no disjoint fuzzy rough A fuzzy rough algebraic closed in (X, TM).

Proposition 1.3.4

If B and D are fuzzy rough somewhere TM dense in a fuzzy rough utra TM connected system (X, TM), then $\mathcal{F}_{\mathcal{RTM}} int(B) \nsubseteq \tilde{1} - \mathcal{F}_{\mathcal{RTM}} int(D)$, in (X, TM).

Proof

Let B *and* D be any two fuzzy rough somewhere TM dense algebras in (X, TM). Then, $\mathcal{F}_{\mathcal{RTM}} intcl(B) \neq \tilde{0}$ and $\mathcal{F}_{\mathcal{RTM}} intcl(D) \neq \tilde{0}$, in (X, TM).

Since (X, TM) is a fuzzy rough utra TM connected system, for the fuzzy rough TM open algebras $\mathcal{F}_{\mathcal{RTM}} intcl(B)$ and $\mathcal{F}_{\mathcal{RTM}} intcl(D)$ in (X, TM).

By *Proposition 1.3.1*,

$$\mathcal{F}_{\mathcal{RTM}} cl[\mathcal{F}_{\mathcal{RTM}} intcl(B)] \nsubseteq \tilde{1} - \mathcal{F}_{\mathcal{RTM}} cl[\mathcal{F}_{\mathcal{RTM}} intcl(D)], \text{ in } (X, TM).$$

Now

$$\mathcal{F}_{\mathcal{RTM}} int(B) \subseteq \mathcal{F}_{\mathcal{RTM}} intcl(B) \subseteq \mathcal{F}_{\mathcal{RTM}} cl[\mathcal{F}_{\mathcal{RTM}} intcl(B)]$$ and

$$\mathcal{F}_{\mathcal{RTM}} int(D) \subseteq \mathcal{F}_{\mathcal{RTM}} intcl(D) \subseteq \mathcal{F}_{\mathcal{RTM}} cl[\mathcal{F}_{\mathcal{RTM}} intcl(D)]$$

Then

$$\mathcal{F}_{\mathcal{RTM}} int(B) \subseteq \mathcal{F}_{\mathcal{RTM}} cl[\mathcal{F}_{\mathcal{RTM}} intcl(B)] \nsubseteq \tilde{1} - \mathcal{F}_{\mathcal{RTM}} cl[\mathcal{F}_{\mathcal{RTM}} intcl(D)] \subseteq \tilde{1} - \mathcal{F}_{\mathcal{RTM}} int(D) \text{ in } (X, TM).$$

Hence it follows that $\mathcal{F}_{\mathcal{RTM}} int(B) \nsubseteq \tilde{1} - \mathcal{F}_{\mathcal{RTM}} int(D)$, in (X, TM).

Proposition 1.3.5

If N and M are fuzzy rough somewhere TM dense in fuzzy rough utra TM connected system, then $\mathcal{F}_{\mathcal{RTM}} int(N) \neq \tilde{0}$ and $\mathcal{F}_{\mathcal{RTM}} int(M) \neq \tilde{0}$, in (X, TM).

Proof

Let N and M be any two fuzzy rough somewhere TM dense in (X, TM). Then $\mathcal{F}_{\mathcal{RTM}} int(N) \nsubseteq \tilde{1} - \mathcal{F}_{\mathcal{RTM}} int(M)$, in (X, TM).

Suppose that $\mathcal{F}_{\mathcal{RTM}} int(N) = \tilde{0}$ and $\mathcal{F}_{\mathcal{RTM}} int(M) = \tilde{0}$, in (X, TM). Then $\tilde{0} = \mathcal{F}_{\mathcal{RTM}} int(N) \nsubseteq \tilde{1} - \mathcal{F}_{\mathcal{RTM}} int(M) = \tilde{1} - \tilde{0} = \tilde{1}$.

Then $\tilde{0} \nsubseteq \tilde{1}$, a contradiction. Thus, $\mathcal{F}_{\mathcal{RTM}} int(N) \neq \tilde{0}$ and $\mathcal{F}_{\mathcal{RTM}} int(M) \neq \tilde{0}$, in (X, TM).

Definition 1.3.7

A fuzzy rough algebraic TM system is said to be a fuzzy rough hypo TM connected system if every non- zero fuzzy rough open sub algebraic of (X, TM) is fuzzy rough TM dense in (X, TM).

Proposition 1.3.6

If A is a fuzzy rough somewhere TM dense in a fuzzy rough utra TM connected [but not fuzzy rough hypo TM connected] system (X, TM), then $\mathcal{F}_{\mathcal{RTM}} int(A) \neq \tilde{0}$ and $\mathcal{F}_{\mathcal{RTM}} cl(A) \nsubseteq \tilde{1}$ in (X, TM).

Proof

Let A be a fuzzy rough somewhere TM dense in (X, TM). Then,

$\mathcal{F}_{\mathcal{RTM}}intcl(A) \neq \tilde{0}$ in (X, TM).

Now $\mathcal{F}_{\mathcal{RTM}}intcl[\tilde{1} - \mathcal{F}_{\mathcal{RTM}}cl(A)] = \tilde{1} - \mathcal{F}_{\mathcal{RTM}}clint[\mathcal{F}_{\mathcal{RTM}}cl(A)]$. By hypothesis, (X, TM) is not a fuzzy rough hypo TM connected system and thus for the fuzzy rough algebraic open $\mathcal{F}_{\mathcal{RTM}}intcl(A)$ in (X, TM). $\mathcal{F}_{\mathcal{RTM}}cl[\mathcal{F}_{\mathcal{RTM}}intcl(A)] \neq \tilde{1}$ implies that $\mathcal{F}_{\mathcal{RTM}}intcl\tilde{1} \neq \tilde{0}$. Thus $\tilde{1} - \mathcal{F}_{\mathcal{RTM}}cl(A)$ is a fuzzy rough somewhere TM dense in (X, TM).

Then by *Proposition 5.3.3*, for the fuzzy rough somewhere TM dense algebra A and $\tilde{1} - \mathcal{F}_{\mathcal{RTM}}cl(A)$, $\mathcal{F}_{\mathcal{RTM}}int(A) \neq 0$ and $\mathcal{F}_{\mathcal{RTM}}int[\tilde{1} - \mathcal{F}_{\mathcal{RTM}}cl(A)] \neq \tilde{0}$ in (X, TM). Then, $\mathcal{F}_{\mathcal{RTM}}int(A) \neq \tilde{0}$ and $\tilde{1} - \mathcal{F}_{\mathcal{RTM}}cl(A) \neq \tilde{0}$ and thus $\mathcal{F}_{\mathcal{RTM}}int(A) \neq \tilde{0}$ and $\mathcal{F}_{\mathcal{RTM}}cl(A) \neq \tilde{1}$ in (X, TM).

Proposition 1.3.7

If a fuzzy rough algebraic TM system (X, TM) is a fuzzy rough utra TM connected system, then there is no fuzzy rough algebraic $D(\neq \tilde{0}, \tilde{1})$ which is both fuzzy rough algebraic open and fuzzy rough algebraic closed in (X, TM).

Proof

Suppose that there exists a fuzzy rough TM algebra D which is both fuzzy rough algebraic open and fuzzy rough algebraic closed in (X, TM). Now D is a fuzzy rough TM closed in (X, TM) implies that $\tilde{1} - D$ is a fuzzy rough algebraic open in (X, TM).

Since (X, TM) is a fuzzy rough utra TM connected system, By *Proposition 5.3.1*, for the fuzzy rough algebraic open algebra D and $\tilde{1} - D$ in (X, TM), $\mathcal{F}_{\mathcal{RTM}}cl(D) \nsubseteq [\tilde{1} - \mathcal{F}_{\mathcal{RTM}}cl(\tilde{1} - D)]$, in (X, TM).

Then $\mathcal{F}_{\mathcal{RTM}}cl(D) \nsubseteq \left[\tilde{1} - \left(\tilde{1} - \mathcal{F}_{\mathcal{RTM}}int(D)\right)\right] = \mathcal{F}_{\mathcal{RTM}}int(D)$. This implies that $D = \mathcal{F}_{\mathcal{RTM}}cl(D) \nsubseteq \mathcal{F}_{\mathcal{RTM}}int(D) = D$, in (X, TM). Hence there is no fuzzy rough algebraic D which is both fuzzy rough algebraic open and fuzzy rough algebraic closed.

Proposition 1.3.8

If $\varphi: (X, TM) \to (Y, TM)$ is a fuzzy rough TM continuous function from a fuzzy rough utra TM connected system (X, TM) into a fuzzy rough algebraic TM system (Y, TM), then (Y, TM) is a fuzzy rough utra TM connected system.

Proof

Let S and G be any two non-zero fuzzy rough algebraic closed in (Y, TM). It is claimed that $S \nsubseteq \tilde{1} - G$, in (Y, TM). Suppose that $S \subseteq \tilde{1} - G$, in (Y, TM). Then, $\varphi^{-1}(S) \subseteq \varphi^{-1}(\tilde{1} - G) = \tilde{1} - \varphi^{-1}(G)$.
Since $\varphi: (X, TM) \to (Y, TM)$ is a fuzzy rough TM continuous function, $\varphi^{-1}(S)$ and $\varphi^{-1}(G)$ are fuzzy rough algebraic closed in (X, TM). Thus, $\varphi^{-1}(S) \subseteq \tilde{1} - \varphi^{-1}(G)$ for the fuzzy rough algebraic closed $\varphi^{-1}(S)$ and $\varphi^{-1}(G)$ in (X, TM), a contradiction to (X, TM) being a fuzzy rough utra TM connected system.

Thus, $S \nsubseteq \tilde{1} - G$, for the fuzzy rough algebraic closed S and G in (Y, TM). Hence (Y, TM) is a fuzzy rough utra TM connected system.

1.4 Some Relationships Between Fuzzy Rough Utra TM Connected System and Other Fuzzy Rough Algebraic TM System

Definition 1.4.1

A fuzzy rough algebraic TM system is said to be a fuzzy rough algebraic TM Brown system if for any two non-zero fuzzy rough algebraic open A and B in (X, TM), $\mathcal{F}_{\mathcal{RTM}}cl(A) \subseteq \tilde{1} - \mathcal{F}_{\mathcal{RTM}}cl(B)$ in (X, TM).

Proposition 1.4.1

If a fuzzy rough algebraic TM system (X, TM) is a fuzzy rough utra TM connected system, then (X, TM) is a fuzzy rough algebraic TM Brown system.

Proof

Let D and E be any two non-zero fuzzy rough TM closed algebra in (X, TM). Since (X, TM) is a fuzzy rough utra TM connected system, by *Proposition 5.3.1*, $\mathcal{F}_{\mathcal{RTM}}cl(D) \nsubseteq \tilde{1} - \mathcal{F}_{\mathcal{RTM}}cl(E)$, in (X, TM). This proves that (X, TM) is a fuzzy rough algebraic TM Brown system.

Remark 1.4.1

The fuzzy rough algebraic TM Brown system need not be fuzzy utra TM connected system.

Example 1.4.1

Let $U = \{a, b, c\}$ and let $X_L = \{a\}$ and $X_U = \{a, c\}$ with $X_L \subset X_U$.
Then the Boolean algebra is $\mathcal{B} = \left\{\tilde{0}, \{a\}, \{b\}, \{b, c\}, \{a, c\}, \{a, b\}, \{a, b, c\}\right\}$.
Define $A_L(x): X_L \to I \Rightarrow A_L(a) = 0.8$
$A_U(x): X_U \to I \Rightarrow A_U(a) = 0.9$ and $A_U(c) = 0.3$
and define $B_L(x): X_L \to I \Rightarrow B_L(a) = 0.3$
$B_U(x): X_U \to I \Rightarrow B_U(a) = 0.4$
and $B_U(c) = 0.2$

Then $A = \left\{\left(\frac{a}{0.8}\right), \left(\frac{a}{0.9}, \frac{c}{0.3}\right)\right\}$ and $B = \left\{\left(\frac{a}{0.3}\right), \left(\frac{a}{0.4}, \frac{c}{0.4}\right)\right\}$ the fuzzy rough algebraic of (X, TM). Therefore, the fuzzy rough algebraic TM system is $\{\tilde{0}, \tilde{1}, A, B\}$. The complement of A and B are $A' = \left\{\left(\frac{a}{0.1}\right), \left(\frac{a}{0.2}, \frac{c}{1}\right)\right\}$ and $B' = \left\{\left(\frac{a}{0.6}\right), \left(\frac{a}{0.7}, \frac{c}{1}\right)\right\}$.

Let us define the fuzzy rough algebraic closed as $D = \left\{\left(\frac{a}{0.6}\right), \left(\frac{a}{0.6}, \frac{c}{1}\right)\right\}$ and $E = \left\{\left(\frac{a}{0.5}\right), \left(\frac{a}{0.6}, \frac{c}{1}\right)\right\}$. Then by computation $\mathcal{F}_{\mathcal{RTM}}cl(D) = \left\{\left(\frac{a}{0.6}\right), \left(\frac{a}{0.7}, \frac{c}{1}\right)\right\}$ and $\mathcal{F}_{\mathcal{RTM}}cl(E) = \left\{\left(\frac{a}{0.6}\right), \left(\frac{a}{0.7}, \frac{c}{1}\right)\right\}$. Then $\tilde{1} - \mathcal{F}_{\mathcal{RTM}}cl(E) = \left\{\left(\frac{a}{0.3}\right), \left(\frac{a}{0.4}, \frac{c}{1}\right)\right\}$ which implies, $\mathcal{F}_{\mathcal{RTM}}cl(D) \nsubseteq \tilde{1} - \mathcal{F}_{\mathcal{RTM}}cl(E)$. Hence, (X, TM) is a brown system but not a utra connected system as $D' \subseteq E$.

Remark 1.4.2

It is to be noted that fuzzy rough utra TM connected system is independent of fuzzy rough hypo TM connected system. For, *Example 5.3.1*. (X, TM) is a fuzzy rough utra TM connected system but not a fuzzy rough hypo TM connected system. Also, a fuzzy rough hypo TM connected system need not be a fuzzy utra TM connected system. For, consider the following example.

Example 1.4.2

Let $U = \{a, b, c\}$ and let $X_L = \{a\}$ and $X_U = \{a, c\}$ with $X_L \subset X_U$.

Then the Boolean algebra is $\mathcal{B} = \{\tilde{0}, \{a\}, \{b\}, \{b, c\}, \{a, c\}, \{a, b\}, \{a, b, c\}\}$.

Define $A_L(x): X_L \to I \Rightarrow A_L(a) = 0.8$

$A_U(x): X_U \to I \Rightarrow A_U(a) = 0.9$ and $A_U(c) = 0.3$

and define $B_L(x): X_L \to I \Rightarrow B_L(a) = 0.7$

$B_U(x): X_U \to I \Rightarrow B_U(a) = 0.8$

and $B_U(c) = 0.3$

Then $A = \left\{\left(\frac{a}{0.8}\right), \left(\frac{a}{0.9}, \frac{c}{0.3}\right)\right\}$ and $B = \left\{\left(\frac{a}{0.7}\right), \left(\frac{a}{0.8}, \frac{c}{0.3}\right)\right\}$ the fuzzy rough algebraic of (X, TM). Therefore, the fuzzy rough algebraic TM system is $\{\tilde{0}, \tilde{1}, A, B\}$. The complement of $A' = \left\{\left(\frac{a}{0.1}\right), \left(\frac{a}{0.2}, \frac{c}{1}\right)\right\}$ and $B' = \left\{\left(\frac{a}{0.2}\right), \left(\frac{a}{0.3}, \frac{c}{1}\right)\right\}$. Define $S = \left\{\left(\frac{a}{0.7}\right), \left(\frac{a}{0.6}, \frac{c}{0.3}\right)\right\}$ and $D = \left\{\left(\frac{a}{0.6}\right), \left(\frac{a}{0.7}, \frac{c}{0.3}\right)\right\}$ be fuzzy rough algebraic open. Then the corresponding fuzzy rough algebraic closed $S' = \left\{\left(\frac{a}{0.4}\right), \left(\frac{a}{0.3}, \frac{c}{1}\right)\right\}$ and $D' = \left\{\left(\frac{a}{0.3}\right), \left(\frac{a}{0.4}, \frac{c}{1}\right)\right\}$. By computation $\mathcal{F}_{\mathcal{RTM}}cl(S) = \tilde{1}$. Hence, (X, TM) is a fuzzy rough hypo connected system but not a fuzzy rough utra connected system as $S' \subseteq D$.

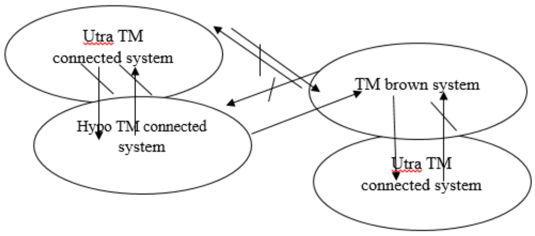
Proposition 1.4.2

If a fuzzy rough TM system (X, TM) is a fuzzy rough utra TM connected system, then (X, TM) is a fuzzy rough TM connected system.

Proof

Let (X, TM) be a fuzzy rough utra TM connected system. Then, by *Proposition 5.3.4*, there is no fuzzy rough algebraic A which is both fuzzy rough algebraic open and fuzzy rough algebraic closed in (X, TM) and hence (X, TM) is a fuzzy rough TM connected system.

The inter relation between fuzzy rough utra TM connected system, fuzzy rough hypo TM connected system, fuzzy rough TM brown system and fuzzy rough TM connected system can be stated as follows in Fig. a:



Definition 1.4.2

A fuzzy rough algebraic TM system is said to be a fuzzy rough algebraic open hereditarily irresolvable system if $\mathcal{F}_{\mathcal{RTM}}intcl(A) \neq \tilde{0}$, for any non-zero fuzzy rough algebraic A defined on X, then $\mathcal{F}_{\mathcal{RTM}}int(A) \neq \tilde{0}$, in (X, TM).

Proposition 1.4.3

If a fuzzy rough algebraic TM system (X, TM) is a fuzzy rough utra TM connected system but not fuzzy rough hypo TM connected system, then (X, TM) is a fuzzy rough algebraic open hereditarily irresolvable system.

Proof

Let H be a fuzzy rough somewhere TM dense algebraic in (X, TM). Since (X, TM) is a fuzzy rough utra TM connected system but not a fuzzy rough hypo TM connected system, $\mathcal{F}_{\mathcal{RTM}}int(H) \neq \tilde{0}$ and $\mathcal{F}_{\mathcal{RTM}}cl(H) \neq \tilde{1}$ in (X, TM). Thus, for the fuzzy rough somewhere TM dense algebraic H, $\mathcal{F}_{\mathcal{RTM}}int(H) \neq \tilde{0}$ implies that (X, TM) is a fuzzy rough algebraic open hereditarily irresolvable system.

Remark 1.4.3

Fuzzy rough algebraic open hereditarily irresolvable system need not be fuzzy rough utra TM connected system.

Example 1.4.3

Let $U = \{a, b, c\}$ and let $X_L = \{a\}$ and $X_U = \{a, c\}$ with $X_L \subset X_U$.

Then the Boolean algebra is $\mathcal{B} = \{\tilde{0}, \{a\}, \{b\}, \{b, c\}, \{a, c\}, \{a, b\}, \{a, b, c\}\}$.

Define $A_L(x): X_L \to I \Rightarrow A_L(a) = 0.3$

$A_U(x): X_U \to I \Rightarrow A_U(a) = 0.5$ and $A_U(c) = 0.3$

and define $B_L(x): X_L \to I \Rightarrow B_L(a) = 0.2$

$B_U(x): X_U \to I \Rightarrow B_U(a) = 0.2$ and $B_U(c) = 0.3$

Then $A = \left\{\left(\frac{a}{0.3}\right), \left(\frac{a}{0.5}, \frac{c}{0.3}\right)\right\}$ and $B = \left\{\left(\frac{a}{0.2}\right), \left(\frac{a}{0.2}, \frac{c}{0.3}\right)\right\}$ the fuzzy rough algebraic of (X, TM). Therefore, the fuzzy rough algebraic TM system is $\{\tilde{0}, \tilde{1}, A, B\}$. The complement of $A' = \left\{\left(\frac{a}{0.5}\right), \left(\frac{a}{0.7}, \frac{c}{1}\right)\right\}$ and $B' = \left\{\left(\frac{a}{0.8}\right), \left(\frac{a}{0.8}, \frac{c}{1}\right)\right\}$. Define fuzzy rough algebraic open $E = \left\{\left(\frac{a}{0.6}\right), \left(\frac{a}{0.7}, \frac{c}{0.2}\right)\right\}$ and $R = \left\{\left(\frac{a}{0.5}\right), \left(\frac{a}{0.5}, \frac{c}{0.3}\right)\right\}$. Then the complement of E and R is $E' = \left\{\left(\frac{a}{0.3}\right), \left(\frac{a}{0.4}, \frac{c}{1}\right)\right\}$ and $R' = \left\{\left(\frac{a}{0.5}\right), \left(\frac{a}{0.5}, \frac{c}{1}\right)\right\}$. Then $\mathcal{F}_{\mathcal{RTM}}int(E) = \left\{\left(\frac{a}{0.3}\right), \left(\frac{a}{0.5}, \frac{c}{0.3}\right)\right\} \neq \tilde{0}$ and $\mathcal{F}_{\mathcal{RTM}}cl(E) = \left\{\left(\frac{a}{0.8}\right), \left(\frac{a}{0.8}, \frac{c}{1}\right)\right\}$. Then $\mathcal{F}_{\mathcal{RTM}}int(\mathcal{F}_{\mathcal{RTM}}cl(E)) = \left\{\left(\frac{a}{0.3}\right), \left(\frac{a}{0.5}, \frac{c}{0.3}\right)\right\} \neq \tilde{0}$. Hence (X, TM) is a fuzzy rough TM open hereditarily irresolvable system need not be fuzzy rough utra TM connected system as $E' \subseteq R$.

Proposition 1.4.4

If $S \nsubseteq \tilde{1} - K$, for any two fuzzy rough nowhere TM dense algebra in a fuzzy rough hypo TM connected system (X, TM), then (X, TM) is not a fuzzy rough utra TM connected system.

Proof

Suppose that S and K are any two fuzzy rough algebraic closed in (X, TM). Then, $\mathcal{F}_{\mathcal{RTM}}cl(S) = S \neq \tilde{1}$ and $\mathcal{F}_{\mathcal{RTM}}cl(K) = K \neq \tilde{1}$ in (X, TM). This implies that S and K are not fuzzy rough TM dense algebraic and S and K are fuzzy rough nowhere TM dense algebraic in (X, TM).

By hypothesis $S \subseteq \tilde{1} - K$. Thus, for the fuzzy rough closed algebraic S and K, $S \subseteq \tilde{1} - K$, implies that (X, TM) is not a fuzzy rough utra TM connected system.

Proposition 1.4.5

If D is a fuzzy rough algebraic closed in a fuzzy rough utra TM connected system but not a fuzzy rough hypo TM connected system (X, TM), then D is a fuzzy rough somewhere TM dense algebraic in (X, TM)

Proof

Let D be a fuzzy rough algebraic TM closed in (X, TM). Since (X, TM) is a fuzzy rough utra TM connected system but not fuzzy rough hypo TM connected system, (X, TM) is a fuzzy rough algebraic open hereditarily irresolvable system. Then, D is a fuzzy rough somewhere TM dense algebra in (X, TM).

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